Freeform Glass Structures
Dr. Ing. Hans Schober, Schlaich Bergermann und Partner, Consulting Engineer, Stuttgart, Germany

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Abstract

With his knowledge of spatial geometry, shell theory and manufacturing process, the structural engineer is able to further develop light and transparent glass roofs. Double curved surface structures with a triangular mesh shape provide favourable prerequisites to single layer membrane shells. This paper describes personal experience with freely shaped glazed shells using triangular and quadrilateral glass panels. A geometric strategy for generating quadrilateral planar facets closely matching the architect’s design is presented.

1 Introduction

Shells possess a natural beauty and efficiency, because the flowing, double-curved shape is able to transfer loads without bending, only in the surface by tension and compression. Only the triangular mesh enables the membrane forces to basically flow in the surface.

Freeform glazed shells matching any design created in physical or digital models are normally triangular meshed and covered by plane triangular glass panels (see chapter 3.3). However, triangulated surfaces are economically less advantageous than equivalent surface structures built of quadrilateral (four sided) facets (fig. 1). Quadrangular mesh constructions require less mullions and less machining operations, because a node with four connections is by far easier to manufacture than one with six, and quadrangular glass panes cost only half as much as triangular ones.

However, this achieved economy can only be maintained if the quadrilateral facets of the surface structure are maintained planar.

![Fig. 1: Shell structure with triangular and quadrangular glass panels](image)

We have obviously solved the antithesis of favourable structural behaviour and difficult, double curved manufacture with our grid shells. The base grid of the structure is manufactured from a quadrangular mesh made of slats, square if laid flat on the ground. This plane mesh can be formed in almost any shape by modifying the original 90° mesh angle. The squares become rhombi. To retain the shell action and obtain the required triangles for the plane’s stiffness, the quadrangular mesh has to be braced diagonally by cables (see fig. 1 top). This design principle was developed in 1989 and 1990 and built first in Neckarsulm and Hamburg (Fig. 2). The geometric principle of the base grid can be traced back to Frei Otto’s grid shells in Montreal and Mannheim and to the cable net roof for the 1972 Olympics in Munich.
Fig. 2: Neckarsulm grid dome, left
   Geometric principle of grid shells, right: a square mesh when laid out into a plane

The previous example show the problem of double curved grid shells since the quadrangles are usually not plane. This is why the thermopanes of the spherical dome in Neckarsulm are spherically curved, resulting in an ideal but expensive spherical shape (fig. 3 bottom). Parts of the free shaped and single glazed transition between the two barrel vaults of the roof in Hamburg had to be dissolved into triangles in areas of extreme warping (fig. 3 top).

Therefore, the construction of double curved shells with quadrangular meshes would be definitely restricted for technical and economical reasons, if there wouldn't be a geometrical technique for designing almost any shape with plane quadrangles be available.
2 Basic geometric principles for planar quadrangles

Having this problem in mind I developed during the last years a geometric strategy for generating quadrilateral planar facets for double-curved surfaces. The method is based on the simple principle that two spatial, parallel vectors are always defining a planar quadangular surface. The vectors and the connection between their points of origin and end points make up the edge of the quadangular surface.

This is not the only method - but a rather simple one - because a planar surface may also be defined by two vectors not running parallel to each other.

Assuming one direction of the quadangular-meshed net to be the sectional curve with its individual sections being the lateral edges, and assuming the other direction with its individual sections being the longitudinal edges and regarding the two lateral resp. longitudinal edges as vectors, then this results in two design principles for plane quadangular mesh:

a) The longitudinal edges of a row of mesh form parallel vectors (Fig. 4 left).
   A special application of this is the translational surface.

b) The lateral edges of a row of mesh form parallel vectors (Fig. 4 right).
   A special application of this is the scale-trans surface.

2.1 The Translational Surface

As published already in 1994 [1] and [2] translational surfaces permit a vast multitude of shapes for grid shells consisting of quadangular planar mesh. Translating any spatial curve (generatrix) against another random spatial curve (directrix) will create a spatial surface consisting solely of planar quadangular mesh (Fig. 5). Parallel vectors are the longitudinal and lateral edges. Subdividing the directrix and the generatrix equally results in a grid with constant bar length and planar mesh.

If, for example, one parabola (generatrix) translates against another parabola (directrix) perpendicular to it, the result will be an elliptical paraboloid with an elliptic layout curve. Two identical parabolas generate a rotational paraboloid with a circular layout curve (Fig. 6). Innovative shapes may be created by adding translational surfaces (Fig. 6).
A directrix curving anticlastically to the generatrix results in a hyperbolic paraboloid, which can also be formed by two systems of linear generatrices (Fig. 7). This allows for the creation of hypar-surfaces with straight edges which are easy to store and may be joined in a variety of combinations. All these examples consist of a grid with constant bar length and planar mesh.

This paper will present only the basic possibilities. The variety in shapes is described in [1] and [2].
2.2 Scale-Trans-Surfaces

The centric expansion of any sectional curve yields a new one with parallel edges (Fig. 8). The center of expansion may be chosen at random. Centric expansion causes each lateral edge vector of the sectional curve to lengthen or shorten by the same factor, while maintaining its direction. Thus from the (n)th sectional curve evolves the similar (n+1)th sectional curve which is then spatially translated – although without rotation – to any given spot (Fig. 9). The longitudinal edges are determined by the line between the points of origin and the end points of the respective lateral edge vectors. The next row of mesh will be created following the same principle with the shape of the new sectional curve depending on the selected expansion factor. Thus row after row is generated.

![Fig. 8: Scaling a curve centrically or eccentricaly results in parallel lateral edges](image)

To obtain homogenous structures analytical curves such as circles, ellipses, hyperbolas or polynomials may be divided equally (identical lateral edge length) and expanded centrically. Translating the center of expansion of the resulting sectional curves along an analytical spatial curve (directrix) and adjusting the translational distance to the expansion factor results in homogenous surfaces with longitudinal edges of the same length in each mesh, and lateral edges of the same length in each sectional curve. Fig. 9 shows a double-curved surface with plane quadrangular mesh created by centrically expanding elliptical curves and translating the center of expansion along a spatially curved directrix. The generatrix and directrix curve in Fig. 10 is composed of randomly chosen curves.

![Fig. 9: Surfaces with plane quadrangular meshes are generated by scaling and translation.](image)

![Fig. 10: Example of a scale-trans surface with plane quadrangular facets](image)
3 Reference Projects by our Consultancy

To demonstrate the different geometric principles some projects of our office have been selected and are described below.

3.1 Translational Surfaces

The principle of an evenly-meshed translational surface with different parabolas as generatrix and directrix was first realized in 1992 with the roof over an elliptical courtyard in Rostock, Germany (Fig. 11).

A parabola (generatrix) translating across another parabola (directrix) perpendicular to it, results in an elliptic layout curve and an evenly-meshed net consisting of plane quadrangular meshes.

The filigree shell consists of solid slats 60mm wide and 40mm deep.

Fig. 11: Gallery Rostocker Hof, Rostock, Germany, grid dome as translational surface

The roof over the square courtyard of the German Historic Museum in Berlin was generated using identical circles as generatrix and directrix. Since the roof is only point supported at the four corners, the grid was oriented in the direction of the flow of forces, that means in diagonal direction.

Due to the small rise, the mullions are 60mm wide and 140mm deep.

Fig. 12: Courtyard roof of the German Historical Museum, Berlin (Arch. I.M. Pei, 2002)

Grid dome as translational surface with planar meshes
However, directrix and generatrix must not necessarily consist of geometrically simple curves but can also be defined as random spatial curves, and thus offer a huge variety of shapes. A recent example is the hippopotamus-house in the Berlin Zoo. Two parabolas with a freely defined transition curves were chosen as a directrix for the roof over two circular ponds (Figs. 13). The double glazing rests directly on mullions only 40mm wide and 40mm deep.

Fig. 13: Glass dome of the Hippo House at the Berlin Zoo as translational surface with planar quadrangular mesh

Another example of designing rather difficult geometries as translational surfaces is the roof over a courtyard in Stuttgart (Bosch Areal), Germany. Although in this case there are several adjoining irregular courtyards, it was possible to create a continuously curved transition area consisting solely of plane and evenly meshed quadrangles with 40mm wide and 60mm deep flat bars (Fig. 14).
3.2 Scale-Trans Surfaces

The plan of the glass roof over the platform of the new Lehrter railroad station in Berlin follows strictly the flaring tracks. The three-centered arch cross-section of the roof, as shown in Fig. 15, allows for optimum adjustment to the clearance and to the flaring tracks. The sectional curve was determined by centric expansion and a slight rotation according the curved tracks, leading to a grid with planar quadrangular meshes. Thus the rise of the hall and the roof’s profile increases with the flaring, an effect which rather enhances the design.
An example for the design of freeform structures is the Jerusalem Museum of Tolerance project currently under design at Gehry Partners in collaboration with our office. This voluminous complex is a multifunctional group of building components, which are each given individual design characters within the context of the overall design (Figure 16). A major component of the project is a series of large, freeform glass structures. One of them, the atrium roof, is described here.

Fig. 16: Computer model of Jerusalem Museum of Tolerance

The challenge here lies not only in the development of a geometric strategy for generating quadrilateral planar facet solutions, but also in the fact that said solutions must closely match the designs created initially in physical model form by the architects. For the atrium roof we used the scale-trans method in order to be able to approximate the translation surface as much as possible to the original designed shape. The control of the surface is achieved by combining a combination of any of the following:
I. Modifying the control points for the directrix curve;
II. Modifying the control points for the generatrix curve;
III. Modifying the scaling law for the whole generatrix curve or only for parts of it.

Due to the constraints of the translation surface, it is not possible to perfectly match the original surface, but the combination of translation and scaling gets quite close.

Fig. 17 shows the result of the form-finding process resulting in a very complex grid shell with planar quadrilateral facets.

Fig. 17: Atrium roof as a scale-trans surface with flat quadrangular facets, matching closely the designers form (Arch. F.O.Gehry, Santa Monica)
3.3 Sculptural shapes

We often face the challenge of designing sculptural or freeform shapes matching perfectly the created physical or digital model form. Since such forms usually can no longer be covered by plane quadrangular panes, the economically less favourable triangular panes have to be used.

The architect F.O. Gehry designed for the DZ Bank in Berlin a three-dimensional barrel shaped atrium roof as a sculpture intervening with the interior (fig. 18).

Free shapes like this can only be glazed with triangles and made from a triangular grid. The shell structure is point supported along the edges at 16m intervals. Due to the barrel like shape the shell had to be additionally stiffened by ‘spoked wheels’ (fig 18 bottom). The entire structure is made of stainless steel. The nodes were milled three-dimensionally.

Fig. 18: DZ Bank’s sculptured roof in Berlin (Arch. F.O. Gehry, Santa Monica)
References
